Study of heat transfers problem of dissipative fluid flow in a porous walls channel

By

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ABSTRACT

A model of heat transfer by natural convection of dissipative fluid in a channel of porous walls has been discussed, the solution of governing partial differential equations obtained using **Alternating Direction Implicit** method. The unsteady state as well as steady state solutions are founds simultaneously during the successive iteration of **(ADI)** for the first time .

دراسة مسألة انتقال الحرارة لمائع جاري قابل للتبدد في قناة مسامية الجدران

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الملخص

لقد تم في هذا البحث دراسة لنموذج انتقال الحرارة بالحمل الحراري الطبيعي لمائع قابل للتبدد في قناة أفقية مسامية الجدران والتي يظهر فيها تأثير الامتصاص على خواص المائع، وقد تمت صياغة المعادلات التفاضلية الجزئية التي تتحكم بالمسألة وتم حلها باستخدام طريقة الفروقات المنتهية Alternating Direction Implicit. حيث تمكنا من الوصول إلى حل المسألة اللازمني باستخدام التكرارات المتتابعة للطريقة أعلاه على المعادلات التي هي بالصيغة المعتمدة على الزمن.

1-Introduction:

The study of fluids flow in a channel of porous walls is very important because it has a wide range of implementation, and this is due to the fact that these flows have many engineering and geophysical applications which include geothermal resources, Blood flow inside human being bodies, building insulation, Oil extraction, heat salt leaching in soils, flow system for transporting lymph , urinary circulatory system, transpiration cooling and many more.

In previous works, Beithou [2] has studied the effect of variable porosity on the free convection flow along a vertical plate embedded in porous medium numerically. The results showed that when porosity increases , temperature variation becomes steepr and the Nusselt number increase utmost linearly with increasing porosity. Al-Odat [3] investigated transient MHD double diffusive of an electrically conducting fluid by free convection over a flat plate embedded in Darcy and non-Darcy porous medium in the presence of surface suction or blowing and magnetic field effects, he was found that the presence magnetic field lowers both the Nusselt and Sherwood numbers in Darcy as well as Fochheimer flow regimes. Makinde [5], presented the unsteady two-dimensional laminar flow of a viscous incompressible and electrically conducting fluid through a channel with the one wall impermeable and the other porous under the influence of a transverse magnetic field, he used the integral method in his investigation. Bukhari [6] has analyzed a linear stability by using the spectral Chebyshev polynomial method to obtain the numerical solution of multi-layer system consisting of the finger convection onset in a fluid layer overlying a porous layer. Mhone [7], presented the investigation of combined effects of a transverse magnetic field and irradiative heat transfer on unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and a non-uniform walls temperature, his results showed that increasing magnetic field intensity reduce wall shear stress while increasing radiation parameter through heat obseption causes an increasing in the magnetic of wall shear stress. Das & Sahoo [8], they considered the unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction when the plate accelerates in its own plane. The governing equations are solved both analytically and numerically using finite difference scheme and finally El-Kabeir [9] discussed an investigation to the thermal dispersion effect on non-Darcy MHD natural convection flow over a permeable sphere maintained at uniform heat flux in a variable porosity porous medium. In this paper, we study the natural convection in a channel with porous walls, the governing differential equations solved using (**ADI**) method.

2-Mathematical Model:

Consider the unsteady flow of a dissipative fluid passing through a long channel with porous walls, the Cartesian coordinate system (x, y, z) has been taken as the x-axis lay in the center of the channel, y-axis represent the width of the channel while the z-axis is the normal of xy plane. Let u, v and w be the velocity components in the directions x, y and z respectively, we assumed that all the components in z direction are vanishes as its illustrated by the figure (2-1).

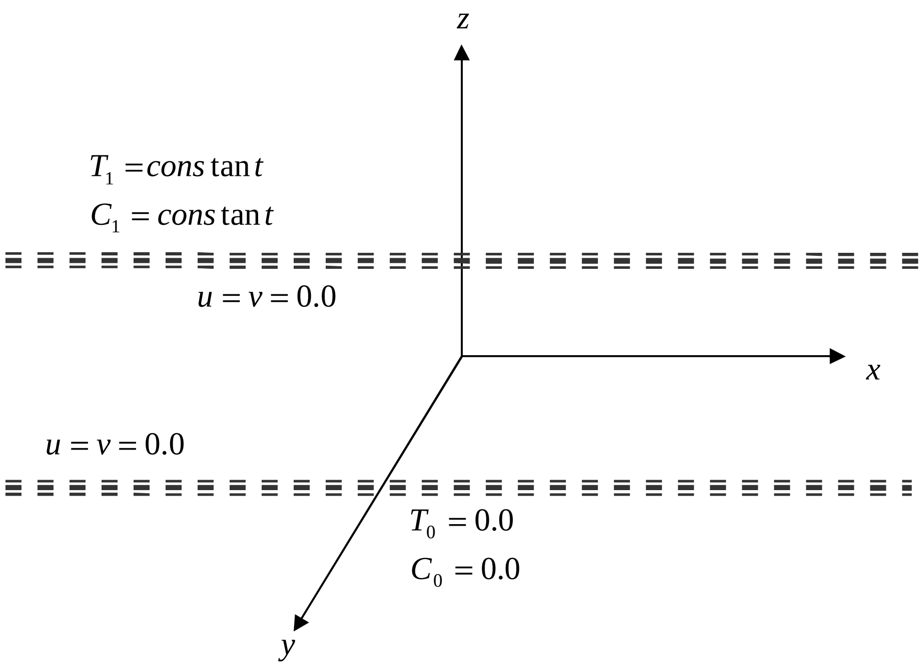

Figure (2-1)

The governing equations in dimensional form are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(2.2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = J\,\nabla^2 v + gb\,(T - T_1) + gb^*(C - C_1) - \frac{J}{k}u \qquad \ldots(2\text{-}2.2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\;\nabla^2 T + \frac{m}{r\,c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2\right] \qquad \ldots(2.2.3)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\nabla^2 C \qquad \ldots(2.2.4)$$

where u, v are the velocity components, t is the time and $T, g, b, b^*, J, k, a, m, r, c_p, D$ are the temperature, gravitational acceleration, thermal expansion coefficient, concentration expansion coefficient, Kinematics viscosity, permeability of the medium, thermal diffusivity, dynamical viscosity, density, specific heat at constant pressure, mass diffusion coefficient respectively.
With the following boundary conditions,

$$\left.\begin{array}{l} u = v = 0.0 \\ T = T_0, T_1 \\ C = C_0, C_1 \end{array}\right\} \qquad y = 0, h \qquad \ldots(2.2.5)$$

h is the width of the channel.

3- Non-dimensional form:

To solve the governing equations (2.2.1)-(2.2.4) with the boundary conditions (2.2.5), we need to introduce the following non-dimensional quantities [4,3],

$$\left.\begin{array}{l} X = \dfrac{x}{h}, \quad Y = \dfrac{y\,Gr^{1/4}}{h}, \quad U = \dfrac{uhGr^{-1/2}}{J}, \quad V = \dfrac{vhGr^{-1/4}}{J} \\ t = \dfrac{tJ\,Gr^{1/2}}{h^2}, \quad pr = \dfrac{J}{a}, \quad q = \dfrac{T - T_0}{T_0 - T_1}, \quad C = \dfrac{C - C_1}{C_0 - C_1} \\ Gr = \dfrac{gb\,h^3 (T_0 - T_1)}{J^2}, \quad Gr^* = \dfrac{gb^* h^3 (C_0 - C_1)}{J^2} \end{array}\right\} \qquad ...(3.1)$$

Substituting these quantities into equations (2.2.1)-(2.2.4), the governing equations becomes.

$$\left[\frac{\sqrt{Gr}\,J}{h^2}\right]\frac{\partial U}{\partial X} + \left[\frac{\sqrt{Gr}\,J}{h^2}\right]\frac{\partial V}{\partial Y} = 0 \qquad \ldots(3.1a)$$

$$\left[\frac{Gr\,J^2}{h^3}\right]\frac{\partial U}{\partial t} + \left[\frac{Gr\,J^2}{h^3}\right]U\,\frac{\partial U}{\partial X} + \left[\frac{Gr\,J^2}{h^3}\right]V\,\frac{\partial U}{\partial Y} = \left[\frac{\sqrt{Gr}\,J^2}{h^3}\right]\frac{\partial^2 U}{\partial X^2} + \left[\frac{Gr\,J^2}{h^3}\right]\frac{\partial^2 U}{\partial Y^2} + \qquad \ldots(3.1b)$$

$$+ g\,b\,(T - T_\infty) + g\,b^*(C - C_\infty) - \frac{\sqrt{Gr}\,J^2}{Kh}U$$

$$\left[\frac{(T_0 - T_1)\sqrt{Gr}\,J}{h^2}\right]\frac{\partial q}{\partial t} + \left[\frac{(T_0 - T_1)\sqrt{Gr}\,J}{h^2}\right]U\,\frac{\partial q}{\partial X} + \left[\frac{(T_0 - T_1)\sqrt{Gr}\,J}{h^2}\right]V\,\frac{\partial q}{\partial Y} =$$

$$= \left[\frac{a\,(T_0 - T_1)}{h^2}\right]\frac{\partial^2 q}{\partial X^2} + \left[\frac{a\,(T_0 - T_1)\sqrt{Gr}\,J}{h^2}\right]\frac{\partial^2 q}{\partial Y^2} + \left(\frac{m}{r\,c_p}\right)\left[\frac{Gr\sqrt{Gr}\,J^2}{h^4}\right]\left(\frac{\partial U}{\partial Y}\right)^2 \qquad \ldots(3.1c)$$

$$\left[\frac{(C_0 - C_1)\sqrt{Gr}\,J}{h^2}\right]\frac{\partial f}{\partial t} + \left[\frac{(C_0 - C_1)\sqrt{Gr}\,J}{h^2}\right]U\,\frac{\partial f}{\partial X} + \left[\frac{(C_0 - C_1)\sqrt{Gr}\,J}{h^2}\right]V\,\frac{\partial f}{\partial Y} = \qquad \ldots(3.1d)$$

$$= D\left[\left(\frac{(C_0 - C_1)}{h^2}\right)\frac{\partial^2 f}{\partial X^2} + \left(\frac{(C_0 - C_1)\sqrt{Gr}}{h^2}\right)\frac{\partial^2 f}{\partial Y^2}\right]$$

Simplifying the above equations, the governing equations under these non-dimensional quantities becomes,

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad ...(3.2)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{1}{\sqrt{Gr}}\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}+q+\frac{Gr^*}{Gr}f-\frac{1}{\sqrt{Gr}Da}U \qquad ...(3.3)$$

$$\frac{\partial q}{\partial t}+U\frac{\partial q}{\partial X}+V\frac{\partial q}{\partial Y}=\frac{1}{\sqrt{Gr}\,pr}\frac{\partial^2 q}{\partial X^2}+\frac{1}{pr}\frac{\partial^2 q}{\partial Y^2}+e\left[\left(\frac{\partial U}{\partial Y}\right)^2\right] \qquad ...(3.4)$$

$$\frac{\partial f}{\partial t}+U\frac{\partial f}{\partial X}+V\frac{\partial f}{\partial Y}=\frac{1}{Sc}\left[\frac{1}{\sqrt{Gr}}\frac{\partial^2 f}{\partial X^2}+\frac{\partial^2 f}{\partial Y^2}\right] \qquad …(3.5)$$

where

Gr = Grashof number for heat transfer

$e=\frac{gb\,h}{C_p}$ = dissipation parameter

pr = Prandtl number

Gr^* = Grashof number for Mass transfer

$Sc=\frac{J}{D}$ = Schmidt number

D = the mass diffusion coefficient

$\mathrm{Da}=\frac{K}{h^2}$ = Darcy number

and the boundary conditions (2.2.5) in the non-dimensional form become,

$$\left.\begin{array}{ll} U=V=0 & at\ Y=0,1 \\ q=0.0 & at\ Y=0 \\ q=10.0 & at\ Y=1 \\ f=0.0 & at\ Y=0 \\ f=1.0 & at\ Y=1 \end{array}\right\} \qquad ...(3.6)$$

4- Method of solution:

In order to solve the system of equations (3.2)-(3.5) with the boundary conditions (3.6), we resort to **ADI** finite difference method [1], and to achieve this we have to start with last equation (3.5), equation of diffusion and then equation (3.4), heat equation, and finally equation (3.3), equation of motion as following:

4-1 Diffusion equation:

$$\frac{f^{*}_{i,j}-f^{n}_{i,j}}{\Delta t/2}+U\frac{f^{*}_{i+1,j}-f^{*}_{i-1,j}}{2\Delta X}+V\frac{f^{n}_{i,j+1}-f^{n}_{i,j-1}}{2\Delta Y}=\frac{1}{Sc}\left[\frac{1}{\sqrt{Gr}}\left(\frac{f^{*}_{i-1,j}-2f^{*}_{i,j}+f^{*}_{i+1,j}}{(\Delta x)^{2}}\right)+\left(\frac{f^{n}_{i,j-1}-2f^{n}_{i,j}+f^{n}_{i,j+1}}{(\Delta y)^{2}}\right)\right] \quad ...(4.1.1)$$

$$\frac{f^{n+1}_{i,j}-f^{*}_{i,j}}{\Delta t/2}+U\frac{f^{*}_{i+1,j}-f^{*}_{i-1,j}}{2\Delta X}+V\frac{f^{n+1}_{i,j+1}-f^{n+1}_{i,j-1}}{2\Delta Y}=\frac{1}{Sc}\left[\frac{1}{\sqrt{Gr}}\left(\frac{f^{*}_{i-1,j}-2f^{*}_{i,j}+f^{*}_{i+1,j}}{(\Delta x)^{2}}\right)+\left(\frac{f^{n+1}_{i,j-1}-2f^{n+1}_{i,j}+f^{n+1}_{i,j+1}}{(\Delta y)^{2}}\right)\right] \quad ..(4.1.2)$$

with boundary conditions,

$$\left.\begin{array}{lll} U = constant & U^{n}_{0,j}=0, & U^{n}_{i,0}=0 \\ V = constant & V^{n}_{0,j}=0, & V^{n}_{i,0}=0 \\ f^{n}_{i,0}=0.0 & f^{n}_{i,N}=1.0 & \end{array}\right\} \quad ...(4.1.3)$$

Equations (4.1.1) and (4.1.2) can be reduced to give,

$$A(I)f^{*}_{i-1,j}+B(I)f^{*}_{i,j}+C(I)f^{*}_{i+1,j}=D_{1}(I), \quad I=0,1,2,...N \quad ...(4.1.4)$$

where

$$\left.\begin{array}{l} A(I)=-\left(U\frac{\Delta t}{2\Delta X}+\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^{2}}\right) \\ B(I)=2\left(1+\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^{2}}\right) \\ C(I)=\frac{U\,\Delta t}{2\Delta X}-\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^{2}} \\ D_{1}(I)=\left(\frac{V\,\Delta t}{2\Delta y}+\frac{\Delta t}{Sc\,(\Delta Y)^{2}}\right)f^{n}_{i,j-1}+2\left(1-\frac{\Delta t}{Sc\,(\Delta Y)^{2}}\right)f^{n}_{i,j}+\left(\frac{\Delta t}{Sc\,(\Delta Y)^{2}}-\frac{V\,\Delta t}{2\Delta Y}\right)f^{n}_{i,j+1} \end{array}\right\} \quad ...(4.1.5)$$

followed by,

$$A_{1}(J)f^{n+1}_{i,j-1}+B_{1}(J)f^{n+1}_{i,j}+C_{1}(J)f^{n+1}_{i,j+1}=D_{2}(J), \quad J=0,1,2,...N \quad ...(4.1.6)$$

where

$$\left.\begin{aligned}
A_1(J) &= -\left(\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta Y)^2} + V\frac{\Delta t}{2\Delta Y}\right)\\
B_1(J) &= 2\left(1+\frac{\Delta t}{Sc\,(\Delta Y)^2}\right)\\
C_1(J) &= \frac{V\,\Delta t}{2\Delta Y} - \frac{\Delta t}{Sc\,(\Delta Y)^2}\\
D_2(J) &= \left(\frac{U\,\Delta t}{2\Delta X} + \frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^2}\right) f^*_{i-1,j} + 2\left(1-\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^2}\right) f^*_{i,j} + \\
&\quad + \left(\frac{\Delta t}{Sc\sqrt{Gr}\,(\Delta X)^2} - \frac{U\,\Delta t}{2\Delta X}\right) f^*_{i+1,j}
\end{aligned}\right\} \quad ...(4.1.7)$$

4-2 Heat equation:

$$\frac{q^*_{i,j} - q^n_{i,j}}{\Delta t/2} + U\frac{q^*_{i+1,j} - q^*_{i-1,j}}{2\Delta X} + V\frac{q^n_{i,j+1} - q^n_{i,j-1}}{2\Delta Y} = \frac{1}{pr\sqrt{Gr}}\left(\frac{q^*_{i-1,j} - 2q^*_{i,j} + q^*_{i+1,j}}{(\Delta x)^2}\right) + \frac{1}{pr}\left(\frac{q^n_{i,j-1} - 2q^n_{i,j} + q^n_{i,j+1}}{(\Delta y)^2}\right) + e\left(\frac{U}{\Delta Y}\right)^2 \quad .(4.2.1)$$

$$\frac{q^{n+1}_{i,j} - q^*_{i,j}}{\Delta t/2} + U\frac{q^*_{i+1,j} - q^*_{i-1,j}}{2\Delta X} + V\frac{q^{n+1}_{i,j+1} - q^{n+1}_{i,j-1}}{2\Delta Y} = \frac{1}{pr\sqrt{Gr}}\left(\frac{q^*_{i-1,j} - 2q^*_{i,j} + q^*_{i+1,j}}{(\Delta x)^2}\right) + \frac{1}{pr}\left(\frac{q^{n+1}_{i,j-1} - 2q^{n+1}_{i,j} + q^{n+1}_{i,j+1}}{(\Delta y)^2}\right) + e\left(\frac{U}{\Delta Y}\right)^2 \quad ...(4.2.2)$$

with boundary conditions,

$$\left.\begin{aligned}
&U = constant \qquad U^n_{0,j} = 0, \quad U^n_{i,0} = 0\\
&V = constant \qquad V^n_{0,j} = 0, \quad V^n_{i,0} = 0\\
&q^n_{i,0} = 0.0 \qquad q^n_{i,N} = 10.0
\end{aligned}\right\} \quad ...(4.2.3)$$

Equations (4.2.1) and (4.2.2) can be reduced to give,

$$A_2(I)q^*_{i-1,j} + B_2(I)q^*_{i,j} + C_2(I)q^*_{i+1,j} = D_3(I), \quad I = 0,1,2,...N \quad ...(4.2.4)$$

where

$$\left.\begin{aligned}
A_2(I) &= -\left(U\frac{\Delta t}{2\Delta X} + \frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2}\right)\\
B_2(I) &= 2\left(1+\frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2}\right)\\
C_2(I) &= \frac{U\,\Delta t}{2\Delta X} - \frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2}\\
D_3(I) &= \left(\frac{V\,\Delta t}{2\Delta y} + \frac{\Delta t}{pr(\Delta Y)^2}\right)q^n_{i,j-1} + 2\left(1-\frac{\Delta t}{pr(\Delta Y)^2}\right)q^n_{i,j} + \left(\frac{\Delta t}{pr(\Delta Y)^2} - \frac{V\,\Delta t}{2\Delta Y}\right)q^n_{i,j+1} +\\
&\quad + e\left(\frac{U}{\Delta Y}\right)^2
\end{aligned}\right\} \quad ...(4.2.5)$$

followed by,

$$A_3(J)q^{n+1}_{i,j-1} + B_3(J)q^{n+1}_{i,j} + C_3(J)q^{n+1}_{i,j+1} = D_4(J), \qquad J = 0,1,2,...N \quad ...(4.2.6)$$

where

$$\left.\begin{aligned}
A_3(J) &= -\left(\frac{\Delta t}{pr(\Delta Y)^2} + V\frac{\Delta t}{2\Delta Y}\right)\\
B_3(J) &= 2\left(1+\frac{\Delta t}{pr(\Delta Y)^2}\right)\\
C_3(J) &= \frac{V\,\Delta t}{2\Delta Y} - \frac{\Delta t}{pr(\Delta Y)^2}\\
D_4(J) &= \left(\frac{U\,\Delta t}{2\Delta X} + \frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2}\right)q^*_{i-1,j} + 2\left(1-\frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2}\right)q^*_{i,j} +\\
&\quad + \left(\frac{\Delta t}{pr\sqrt{Gr}\,(\Delta X)^2} - \frac{U\,\Delta t}{2\Delta X}\right)q^*_{i+1,j} + e\left(\frac{U}{\Delta Y}\right)^2
\end{aligned}\right\} \quad ...(4.2.7)$$

4-3 Motion equation:

$$\begin{aligned}
&\frac{U^*_{i,j} - U^n_{i,j}}{\Delta t/2} + U\frac{U^*_{i+1,j} - U^*_{i-1,j}}{2\Delta X} + V\frac{U^n_{i,j+1} - U^n_{i,j-1}}{2\Delta Y} = \frac{1}{\sqrt{Gr}}\left(\frac{U^*_{i-1,j} - 2U^*_{i,j} + U^*_{i+1,j}}{(\Delta x)^2}\right) +\\
&\qquad + \left(\frac{U^n_{i,j-1} - 2U^n_{i,j} + U^n_{i,j+1}}{(\Delta y)^2}\right) + q^{n+1}_{i,j} + \frac{Gr^*}{Gr}f^{n+1}_{i,j} - \frac{1}{\sqrt{Gr}Da}U
\end{aligned} \quad ...(4.3.1)$$

$$\frac{U_{i,j}^{n+1}-U_{i,j}^{*}}{\Delta t/2}+U\frac{U_{i+1,j}^{*}-U_{i-1,j}^{*}}{2\Delta X}+V\frac{U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}}{2\Delta Y}=\frac{1}{\sqrt{Gr}}\left(\frac{U_{i-1,j}^{*}-2U_{i,j}^{*}+U_{i+1,j}^{*}}{(\Delta x)^2}\right)+ \\ +\left(\frac{U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}}{(\Delta y)^2}\right)+q_{i,j}^{n+1}+\frac{Gr^*}{Gr}f_{i,j}^{n+1}-\frac{1}{\sqrt{Gr}Da}U \qquad ...(4.3.2)$$

with boundary conditions,

$$\left.\begin{array}{lll} U=constant & U_{0,j}^{n}=0, & U_{i,0}^{n}=0 \\ V=constant & V_{0,j}^{n}=0, & V_{i,0}^{n}=0 \end{array}\right\} \qquad ...(4.3.3)$$

Equations (4.3.1) and (4.3.2) can be reduced to give,

$$A_4(I)U_{i-1,j}^{*}+B_4(I)U_{i,j}^{*}+C_4(I)U_{i+1,j}^{*}=D_5(I), \quad I=0,1,2,...N \qquad ...(4.3.4)$$

where

$$\left.\begin{array}{l} A_4(I)=-\left(U\frac{\Delta t}{2\Delta X}+\frac{\Delta t}{\sqrt{Gr}(\Delta X)^2}\right) \\ B_4(I)=2\left(1+\frac{\Delta t}{\sqrt{Gr}(\Delta X)^2}\right) \\ C_4(I)=\frac{U\,\Delta t}{2\Delta X}-\frac{\Delta t}{\sqrt{Gr}(\Delta X)^2} \\ D_5(I)=\left(\frac{V\,\Delta t}{2\Delta y}+\frac{\Delta t}{(\Delta Y)^2}\right)U_{i,j-1}^{n}+2\left(1-\frac{\Delta t}{(\Delta Y)^2}\right)U_{i,j}^{n}+\left(\frac{\Delta t}{(\Delta Y)^2}-\frac{V\,\Delta t}{2\Delta Y}\right)U_{i,j+1}^{n}+ \\ \qquad +\Delta t\,q_{i,j}^{n+1}+\frac{\Delta t\,Gr^*}{Gr}f_{i,j}^{n+1}-\frac{\Delta t}{\sqrt{Gr}Da}U \end{array}\right\} \qquad ...(4.3.5)$$

followed by,

$$A_5(J)U_{i,j-1}^{n+1}+B_5(J)U_{i,j}^{n+1}+C_5(J)U_{i,j+1}^{n+1}=D_6(J), \quad J=0,1,2,...N \qquad ...(4.3.6)$$

where

$$\left.\begin{aligned}
A_5(J) &= -\left(\frac{\Delta t}{(\Delta Y)^2} + V\frac{\Delta t}{2\Delta Y}\right)\\
B_5(J) &= 2\left(1+\frac{\Delta t}{(\Delta Y)^2}\right)\\
C_5(J) &= \frac{V\,\Delta t}{2\Delta Y} - \frac{\Delta t}{(\Delta Y)^2}\\
D_6(J) &= \left(\frac{U\,\Delta t}{2\Delta X} + \frac{\Delta t}{\sqrt{Gr}\,(\Delta X)^2}\right)U^*_{i-1,j} + 2\left(1-\frac{\Delta t}{\sqrt{Gr}\,(\Delta X)^2}\right)U^*_{i,j} + \\
&\quad + \left(\frac{\Delta t}{\sqrt{Gr}\,(\Delta X)^2} - \frac{U\,\Delta t}{2\Delta X}\right)U^*_{i+1,j} + \Delta t\, q^{n+1}_{i,j} + \frac{\Delta t\, Gr^*}{Gr} f^{n+1}_{i,j} - \frac{\Delta t}{\sqrt{Gr}\,Da}U
\end{aligned}\right\} \quad \ldots(4.3.7)$$

The coefficients U,V are treated as constants during any one time-step of the computation [4], each of the equations (diffusion, heat, motion) creating a tridiagonal system which are solved by using Gauss elimination method, all details are given in reference no. [1].

5- Results and figures:

We present in this section some the results obtained from the computation done, and these results have been expressed by figures to illustrates how the solution for different cases becomes as well as the effects of different parameters as the following,

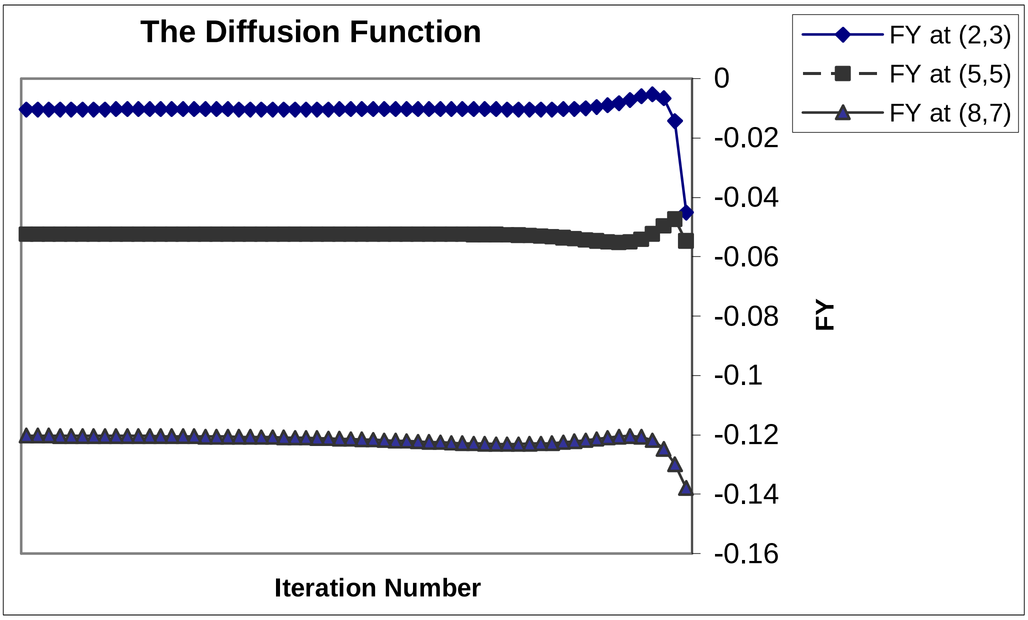

Figure (5.1) The non-dimensional diffusion function f for different position in the channel with the parameters: $Gr = 0.1$, $Sc = 0.22$

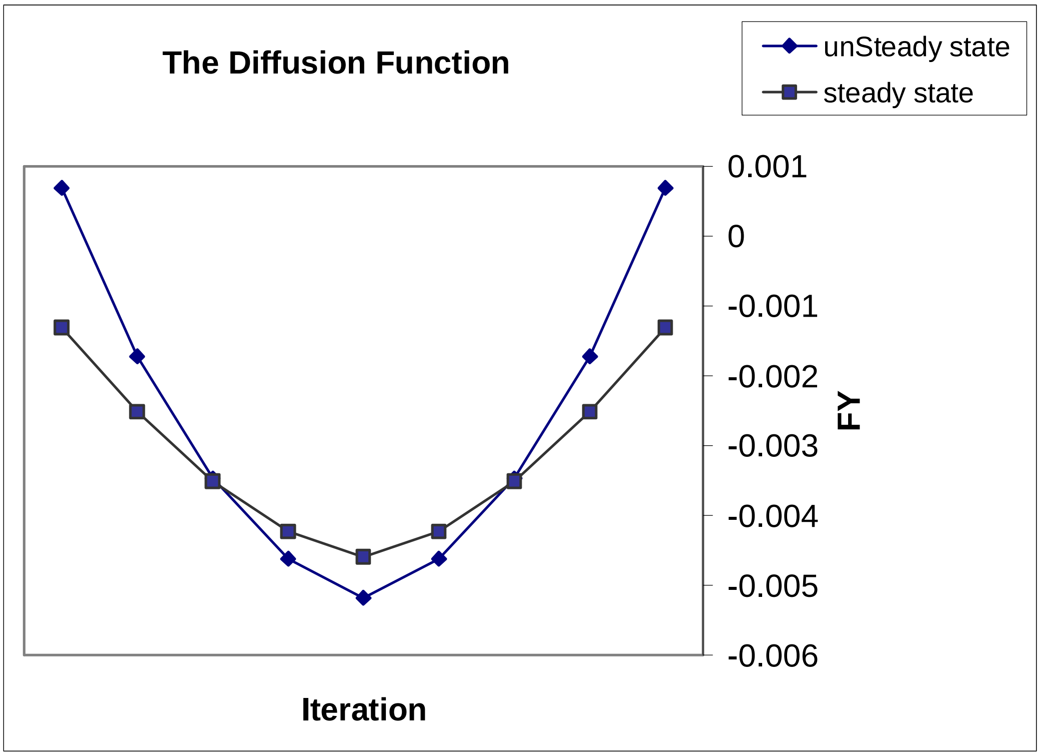

Figure (5.2) The non-dimensional diffusion function f near the side of the channel with the parameters $Gr = 0.1$, $Sc = 0.22$

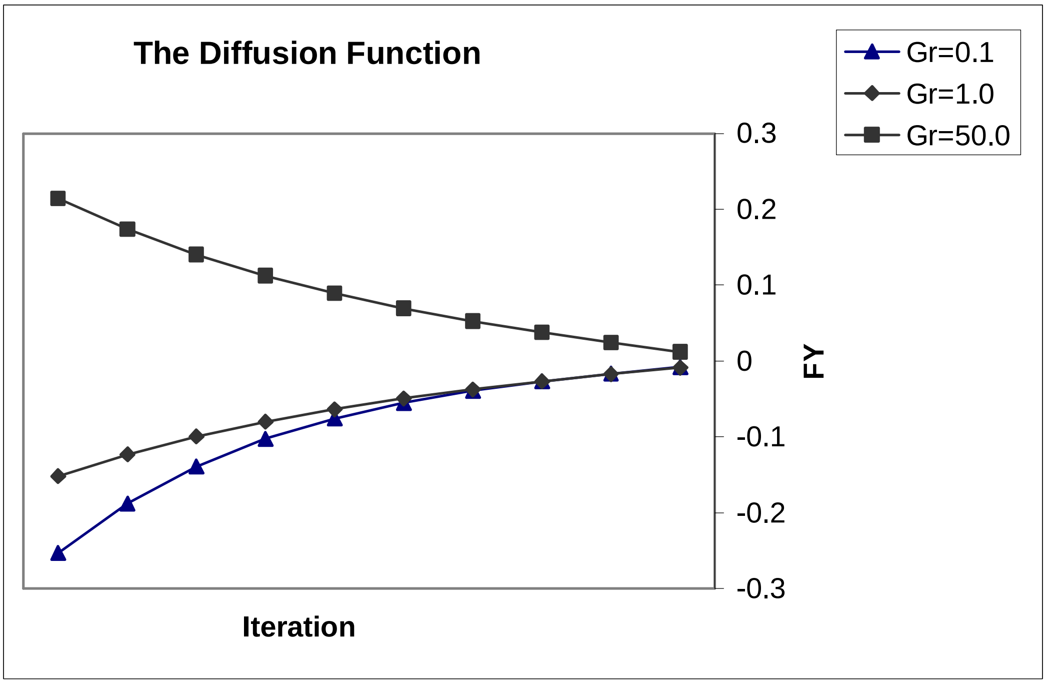

Figure (5.3) The non-dimensional diffusion function f for a point in the channel with the different values of Grashof number .

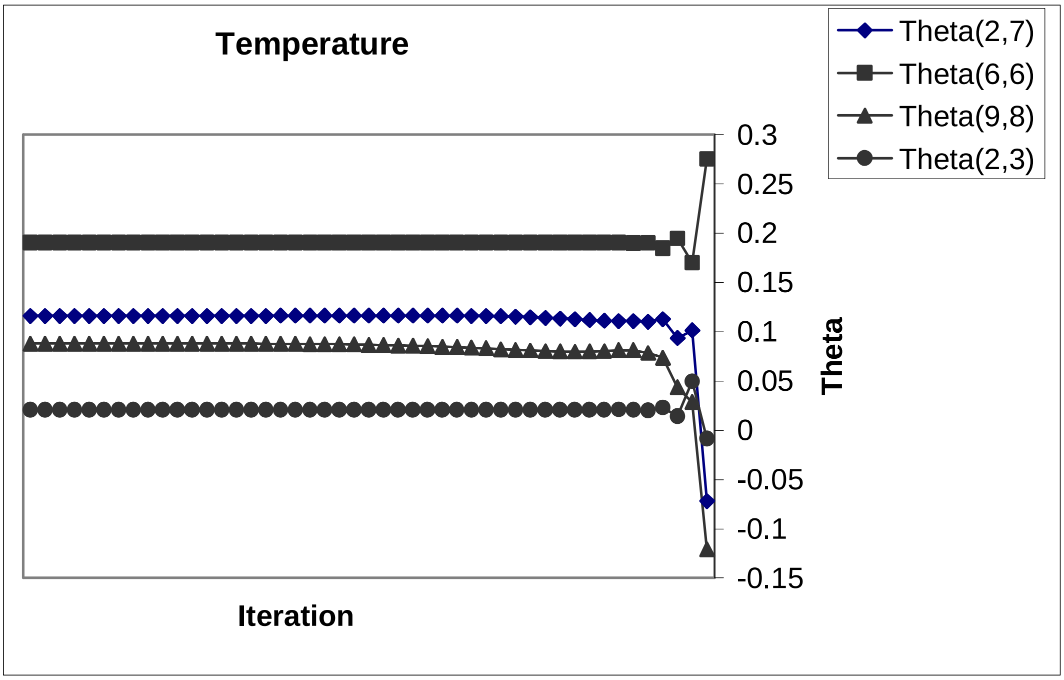

Figure (5.4) The non-dimensional temperature q for different position in the channel with the parameters: $Gr = 0.1$, $pr = 0.7$, $e = -0.004$

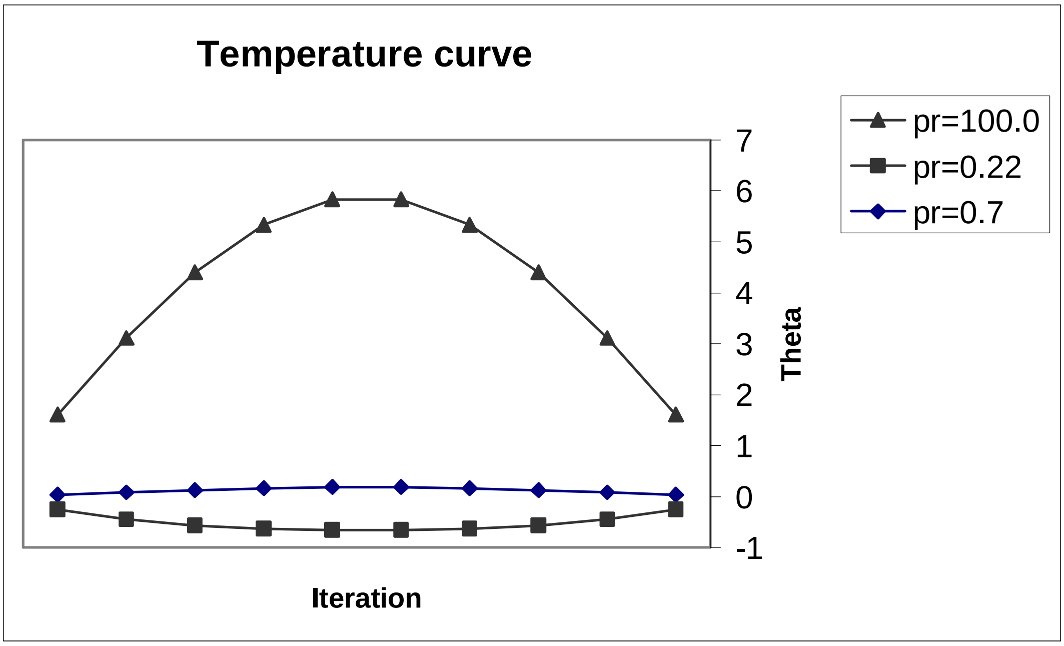

Figure (5.5) The non-dimensional temperature q for a point in the channel with the different values of Prandtl number .

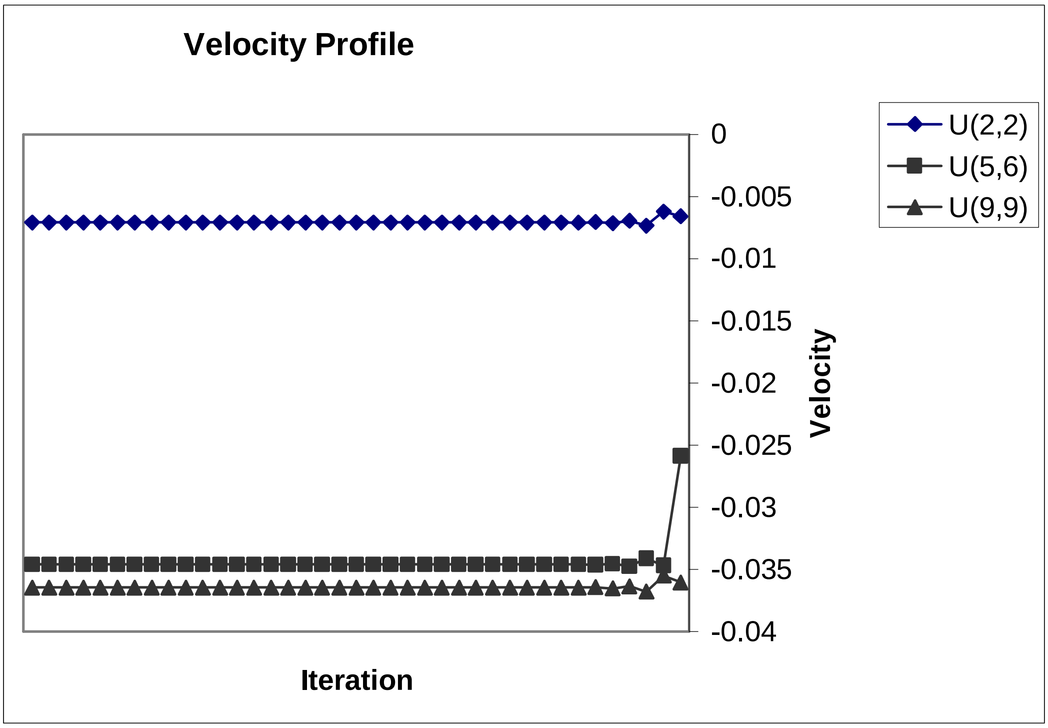

Figure (5.6) The non-dimensional velocity u for different position in the channel with the parameters: $Gr = 0.1$, $Gr^* = 1.0$, $Da = 4.0$

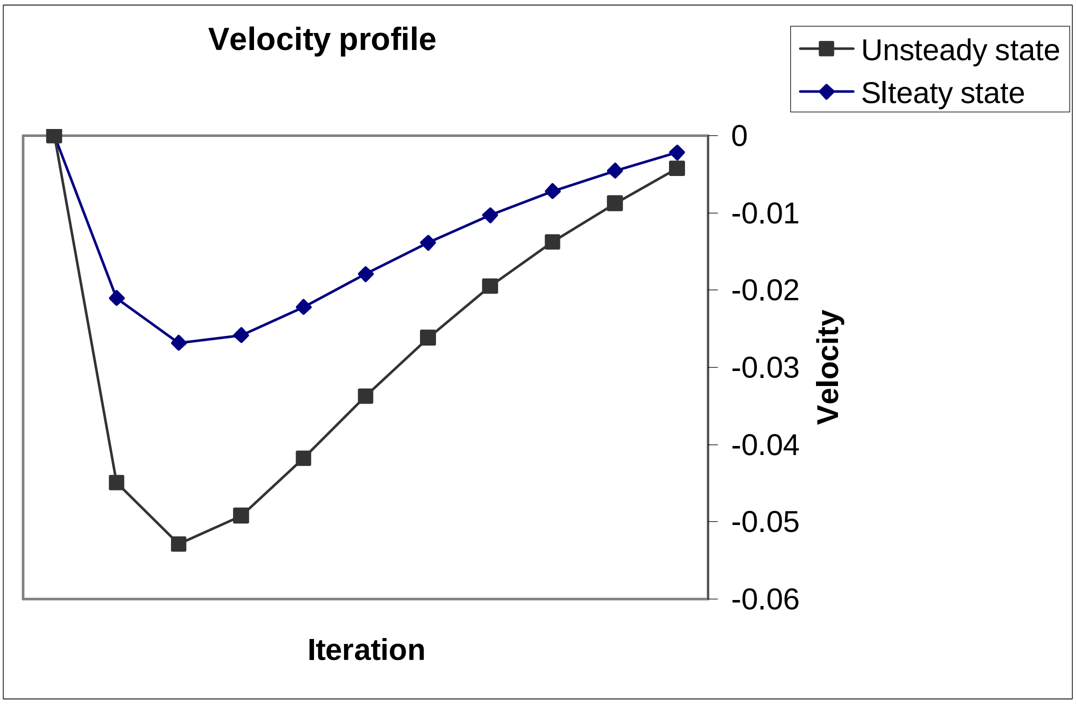

Figure (5.6) The non-dimensional velocity u near the side of the channel with the parameters: $Gr = 0.1$, $Gr^* = 1.0$, $Da = 4.0$

6- Conclusions:

In this work we have used *ADI* method in the solution of the governing equations completely without of reducing or changing and from the results obtained we conclude that the steady state can be reached after some iterations for all equations and this is clear from the figures given previous by last section, due to this fact figure (5.1) represent the results obtained by the solution of diffusion equation which showed that for different point the values of the diffusion function goes to steady state for some iterations and remain fixed to the end, same things happened to the temperature and velocity functions which appears from figures (5.4) and (5.5) respectively. Another remarks can be noticed for some parameters like grashof and prandtl numbers. It is also noticed that the parameters Sc (Schmidt number) and pr (Prandtl number) has effects into motion equation only through diffusion factor f and heat factor q equation (3.3).

7- References:

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