Digital Image Watermarking Using Singular Value Decomposition

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ABSTRACT

This paper presents a digital image watermarking that applied the theory of linear algebra called “singular value decomposition (SVD)” to digital image watermarking. SVD watermarking scheme, which successfully embeds watermarks in to images and its invisible watermarks. SVD method can transform matrix A into product $USV^T$.

Watermarking, is the process of embedding data into a multimedia element, can be used primarily for copyright protection and other purposes. The schemes that have recently been proposed modify the pixel values or Transform domain coefficients. The Singular Value Decomposition (SVD) is a practical numerical tool with applications in a number of signal processing fields including image compression. In an SVD-based watermarking scheme, the singular values of the cover image are modified to embed the watermark data. This method has been proposed An optimal SVD-based watermarking scheme that embeds the watermark in two steps. In the first step, the cover image is divided into smaller blocks and a piece of the watermark is embedded in each block. In the second step, extracting the watermark from the embedded digital image.

All tests and experiments are carried out by using MATLAB as computing environment and programming language.

Keywords: Watermarking, Singular Value Decomposition, Digital Watermarking, Image Processing
1. INTRODUCTION

In recent days, usage of computer networks for communication and for information sharing leads to increase in size of Internet. As the size of the Internet grows, the volume of multimedia data (images, text, video / audio) floating around also increases day by day. As many advanced tools are readily available to duplicate and modify those data in the Internet easily, security is the major concern, which requires some mechanisms to protect digital multimedia data. Thus watermarking is a technique which supports with feasible solution.

Image processing is computer imaging where the application involves a human being in the visual loop . in other words, the image are to be examined and acted upon by people .An image can be defined as a two dimension function \( f(x, y) \) (2D image), where \( x \) and \( y \) are spatial coordinates, and the amplitude of \( f \) at any pair of \((x, y)\) is gray level of the image at that point. For example, a grey level image can be represented as:

\[
 f_{ij} \quad \text{Where} \quad f_{ij} = f(x_i, y_j)
\]

When \( x, y \) and the amplitude value of \( f \) are finite, discrete quantities, the image is called “a digital image”.[1,2]

Watermarking (data hiding) is the process of embedding data into a multimedia element such as image, audio or video. This embedded data can later be extracted from, or detected in, the multimedia for several purposes including copyright protection, access control and broadcast monitoring.

Digital Watermarking is defined as the process of hiding a piece of digital data in the cover data which is to be protected and extracted later for ownership verification.[3,4]

A digital watermarking can be visible or invisible . A visible watermark typically consists of a conspicuously visible message or a company logo indicating the ownership of the image . On the other an invisibly watermarked image appears very similar to the original . The existence of an invisible watermark can only be determined using an appropriate watermark extraction or detection algorithm.[5]

A watermarking algorithm consists of the watermark structure, an embedding algorithm and an extraction, or a detection, algorithm. Watermarks can be embedded in the pixel domain or the transform domain such as the DCT or wavelet. In most multimedia applications, three desired attributes for a
watermarking scheme are invisibility, robustness and high capacity. Invisibility refers to the degree of distortion introduced by the watermark and its affect on the viewers or listeners. Robustness is the resistance of an embedded watermark against intentional attacks, and normal audio/visual processes such as noise, filtering, resembling, scaling, rotation, cropping and loss compression. [6]

A basic image watermarking algorithm consists of a cover image, a watermark structure, an embedding algorithm, and an extraction algorithm. A variety of watermarking techniques have been proposed for multimedia protection, and in particular for digital images. [7,8,9] These techniques can be divided into two main categories according to the embedding domain of the cover image: spatial domain methods and transform domain methods [9]. The spatial domain methods are the earliest and simplest watermarking techniques but have a low information hiding capacity, and also the watermark can be easily erased by lossy image compression. On the other hand, the transform domain approaches insert the watermark into the transform coefficients of the image cover, yielding more information embedding and more robustness against watermarking attacks. [7,8,9]

Singular value decomposition is a numerical technique used to diagonalize matrices in numerical analysis. It is an algorithm developed for a variety of applications.

Any matrix 'M' is decomposed into three sub matrices [u, s, v] such that: 
\[ M = u^T s v \]
Where ‘u’ and ‘v’ are the orthogonal matrices such that \( u^T u = I \) and \( v^T v = I \) where ‘I’ is the Identity matrix and ‘s’ is the diagonal matrix \( \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_N \end{pmatrix} \) such that \( s_1 \geq s_2 \geq s_3 \ldots \geq s_{(N-1)} \geq s_N \). [10]

These values are known as singular values, and matrices u and v are known as corresponding singular vectors [11]. The above decomposition is termed as Singular Value Decomposition.

A SVD, applied to the image matrix, provides singular values (diagonal matrix’s) that represent the luminance or color intensity of the image while the matrices ‘u’ and ‘v’ represents the geometry of the image. It has been scientifically proved that slight variation in the singular values doesn’t change the visual perception of the image. [10]

In past years, several SVD-(Singular Value Decomposition) based digital image watermarking schemes have been proposed in SVD-based digital image watermarking using complex wavelet transform at 2009. And An Optimal Watermarking Scheme Based On Singular Value Decomposition. SVD is one
of the most useful tools of linear algebra with several applications in digital image watermarking [5,6,7,9,10,11,12]

In Santhi and Dr. Arunkumar (2009) DWT_SVD combined based technique is proposed for hiding watermark in full frequency band of color image .

In this paper implements the SVD . so that the hidden digital image watermarks can be removed and high image quality can be provided in restored images.

Watermark has been embedded to the original image .In first ,the host image is divided into smaller blocks then SVD of blocks ,variable scaling factor and distributed over the image blocks .In the second the image are scaled by using a constant scaling factor and combined with watermark image then the values are substituted with the original image.

MATLAB is used as a platform of programming and experiments in this project, since MATLAB is a high-performance in integrating computation, visualization and programming.

The rest of this paper is organized as follows. In Section 2, we describe theory of singular value decomposition SVD. Several experimental results are illustrated and discussed in Section 3. Finally, conclusion and future work are stated in Section 5.

2. THEORY OF SINGULAR VALUE DECOMPOSITION
2.1 SINGULAR VALUE DECOMPOSITION

Singular value decomposition (SVD) is a linear algebra technique that decomposes a given matrix into three component matrices [15]: (1) the left singular vectors; (2) a set of singular values; and (3) and right singular vectors. The two matrices that are made up of singular vectors provide information about the structure of the original matrix. The singular Values describe the strength of the given components of the original matrix. The SVD theorem [13] states that given a matrix M, then there exists a decomposition of M such that \( A=USVT \) see the figure 1 for illustration .

The SVD of a matrix can also be described geometrically. The SVD shows that the values of any matrix A can be reconstructed by a rotation (U), followed by increasing the matrix values (S),followed by another rotation (V) [16]
Figure 1: Illustration of Factoring $A$ to $USVT$. [19]

For example, if $A$ represented coordinates that generated a three-dimensional shape, then that shape could be constructed from the rotational information in $U$ and $V$, along with stretching the shape out to its proper size with the information in $S$ [16]. This type of decomposition can be important and useful in that the rotational matrices isolate the key components of the original matrix, finding relationships between the various data points, while the strength matrix indicates which of the key components illuminated in the rotational matrices are the most important [15, 16, 17]. In our research, this core idea of isolating key components of the original matrix is the basis for using the SVD. When the matrix is comprised of change records, fault information, or some other data from the development process, these key components highlight underlying structures in the code base. The SVD has other uses in computing. For instance, this technique can be used in Image and signal compression. A gray scale image could be represented as a two dimensional matrix made up of intensity values, indicating the darkness of a particular pixel. In this instance, we could treat the image matrix itself as the original $M$ matrix and perform a SVD on it. Once the decomposition is completed, the resulting matrices, $USVT$, can also be represented as the sum of component matrices, as shown in Equation

$$M = u_1 s_{1,1} v_1^T + u_2 s_{2,2} v_2^T + \ldots + u_k s_{k,k} v_k^T$$

(1)

Where $k$ is less than or equal to the rank of matrix $M$. This provides a rank-$k$ approximation of the matrix. The first component of this factorization indicates the component that has the largest singular value and contributes the most variability to the overall matrix. As subsequent components are added together, the matrix, and thus the image itself, begins to re-take its original form [15, 17].
2.2 Singular Value Decomposition watermarking

SVD watermarking is designed to work on binary. For an image of \( N \times N \) pixels and a binary watermark of \( p \times p \) pixels, they first divided the image into \( (N/4) \times (N/4) \) non-overlapping blocks whose size is \( 4 \times 4 \) pixels. Which is based to decide the positions of embedded blocks for each watermark bit. The watermarking embedding procedure show in this figure.

![Figure 2: Watermark embedding procedure](image)

Using SVD on different blocks of the image. For each generated \( S_j \) matrix of each block \( B_j \), where \( 1 \leq j \leq \left( \frac{N}{4} \right) \times \left( \frac{N}{4} \right) \), as below, they let \( s_3 \) be equal to \( s_2 \) and set \( s_2 \) be equal to \( s_2 + \alpha \times w_i \) for embedding the binary value of the watermark, where \( \alpha \) is a constant and \( w_i \) is the watermark bit. Each \( S_j \) matrix of a block \( B_j \) contains value of the watermark. After embedding watermark into \( S_j \) matrix of block \( B_j \), the embedded block \( B'_j \) is obtained by inversing its corresponding \( U, V \).

The proposed watermarking can be broken into two procedures: embedding, and extracting and restoring.[20].

2.3 properties of the SVD

There are many properties and attributes of SVD, here we just present parts of the properties that we used:

1. The singular values \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are unique, however, the matrices \( U \) and \( V \) are not unique;
2. Since \( A^T A = V S^T S V^T \), so \( V \) diagonalizes \( A^T A \), it follows that the \( V_j \) s are the eigenvector of \( A^T A \)
3. Since \( A A^T = U S S^T U^T \), so it follows that \( U \) diagonalizes \( A A^T \) and that the \( U_i \)'s are the eigenvectors of \( A A^T \).
4. If A has rank of r then vj, vj, ..., vr form an orthonormal basis for range space of $A^T$, $R(A^T)$, and uj, uj, ..., ur form an orthonormal basis for range space $A$, $R(A)$.

5. The rank of matrix A is equal to the number of its nonzero singular values .[19].

2.4 Implementation of Singular Value Decomposition (SVD)

The watermark is the signal which is actually added to the cover image $A( k \times k)$. A cover image $A$ is divided into $n \times n$ blocks. For each $n \times n$ block, $\lambda_{\text{max}}$ denotes the largest SVD of the cover image. where $r$, the rank of $W$, is less than or equal to $l$. The order of the SVs of $W$ is randomized, and each $\lambda_{wi}$ is embedded into one $n \times n$ block of the cover image.

2.4.1 Steps of the embedding watermark

1. Input a cover image of size $N \times N$ and watermark image $W$.
2. Convert the RGB components of color image to YCbCr. luminance (Y) and chrominance (Cb and Cr) color values as columns, rgbmap is returned as an M-by-3 matrix that contains the Red, Green, and Blue values equivalent to those colors.
3. The SVD technique is applied on each segment of cover data image as well as on watermark using $[U S V]=\text{SVD}(\text{segment})$, $U, V$ matrices, $S$ is the diagonal matrix.
4. Combine the watermark with the $SVD$ of the selected segment using appropriate scaling factor; $\alpha$, as illustrated in this Equation

$$D = S + \alpha \times W$$

(2)

5. Inverse SVD transform technique is applied to get the watermarked image[14]. See the figure 3 and figure 4 Illustration the Watermark embedding.
2.4.2 Extraction steps as follow:

In the extracting and restoring procedure, not only the hidden watermark must be extracted correctly from the watermarked image, but the watermarked image must also restored with high image quality after the hidden bits are extracted. The watermarking is extracted using

$$ W = (D - S)/\alpha $$

Where $D$ as the owner of the watermarked image subtract the original matrix divided by the value of scale factor ($\alpha$). See in this figure Illustration the Watermark extraction.

Figure 4: diagram of Watermark embedding

Figure 5: Extracting Watermark from an Image
Figure 6: Flow chart of embedding digital image watermark

- Start
- Read Image Data
- Read digital Image watermark
- Read The Value of Scale Factor
- If (reading image) RGB
  - YES: convert YCbCr
  - NO: Divided The Original Image to Segments According of Size Image Data Reading
- Implementation The Singular Value Decomposition (SVD) technique
  - \( A = U S V^T \)
- Select Location to Embedding
- Digital Image Watermark Embedding using \( D = S + wi \)
- Push Embedding In segments original Data
- Invers Singular Value Decomposition of Segment embedding
- Display the Original Image with Digital Image Watermarked embedding
- Calculate the PSNR (Peak Signal to Noise Ratio)
  - \( MSE = \frac{1}{MN} \sum_{ij}(xy_{ij} - wz_{ij})^2 \)
  - \( PSNR = 10 \times \log_{10} \frac{255^2}{MSE} \)
- End
Figure 7: Flow chart of extraction digital image watermark
3. RESULTS

Figure 8(a) shows an example, a 256 × 256 grayscale image Lena is used as the cover image. Figure 8(b) is used as the watermarked image of size 32×32. Figure 8(c) shows the Lena image of size 512 × 512 is taken as cover image and Figure 8 (d) shows the AAA .Tif image of size 256×256. Figure 8(e) shows an example image in Figure 8(e).

3. RESULTS

Figure 8: (a) Original Image                                 (b) Watermark Image
Figure 8: (c) Lena (color)                   (d) AAA.tif                           (e) Blubridge.bmp

Test case is being simulated by taking Lena 256 × 256 grayscale image (shown in Figure 8 (a)) taken a cover image and the watermark image is of size 32 × 32, which is also grayscale is hiding in original image. Figure 9 show the original image and watermarked embedding in the cover image(Lena) using by the following equation

\[ A=USV^T \]
\[ D=S+\alpha W \]

The value of the constant ‘\( \alpha \)’ used is 0.001.
The quality of the watermarked image is measured through PSNR (‘Peak Signal to Noise Ratio’) and the MSE ('Mean Square Error') which is defined as below:

\[ \text{MSE} = \frac{1}{NM} \sum (k_{ij} - \text{host}_{ij}) \]  

\[ \text{PSNR} = 10 \log \left( \frac{R^2}{\text{MSE}} \right) \]

In the previous equation, \( M \) and \( N \) are the number of rows and columns in the input images, respectively. Then the block computes the PSNR using the following equation

The PSNR block computes the peak signal-to-noise ratio, in decibels, between two images. This ratio is often used as a quality measurement between the original and a compressed image.

The Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR) are the two error metrics used to compare image compression quality. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error. The lower the value of MSE, the lower the error.
Figure 10(1,2,3 and 4) shows that inserting the watermarked image and the calculated PSNR value. Also, the extracted watermark is exactly similar to the original watermark image. The performance of the proposed technique is tested in Matlab software version 7.4.

![Figure 10: Reading Gray scale Lena Image](image)

PSNR=46.4357  
MSE=1.4774
Figure 10: Reading Color Lena Image

PSNR = 43.9351
MSE = 2.6277
Figure 10: Reading AAA .Tif Image

Figure 10: Reading Gray Scale Bridge Image
4. CONCLUSION AND FUTURE WORK

This project has applied technique of linear algebra “singular value decomposition (SVD)” to digital image processing. The proposed algorithm depends on embedding the watermark into SVD of original image after dividing it into blocks. The experimental results show the proposed given fidelity and against noise, and compression. We found that SVD is a stable and effective method to split the system into a set of linearly independent components, each of them is carrying own data (information) to contribute to the system. SVD has the advantage of providing a good method of embedding watermark.

In the future work, the detection system will be extend to more transform domain watermarking approaches such as DWT-SVD and DCT-SVD. The application can be performed programming of C++ or Visual Basic.

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